Math 20E - Lecture A00	**Midterm #2, VERSION C**
Fall 2016	**Name:** _____
11/18/2016	PID: _____
Time Limit: 50 Minutes	Section Time: _____

This exam contains 1 pages (including this cover page) and 4 questions.
Total of points is 100.

You may not use any notes (except your cheat sheet) or calculators during this exam.
Write your *Name, PID, and Section* on the front of your Blue Book.
Write the *Version* of your exam on the front of your Blue Book.
Write your solutions clearly in your Blue Book.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.

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1. (25 points) Evaluate $\int_C y\,dx - x^2\,dy$, where C is the boundary of the square $[-1,1]\times[-1,1]$ oriented in the counterclockwise direction, *using Green's theorem.*

2. (25 points) Let $\Phi(u,v) = (u^2, u\cos v, u\sin v)$ be a parametrized surface, where $0 \le u \le 3$ and $0 \le v \le 2\pi$.

   a. (10 points) Compute the tangent vectors $\mathbf{T}_u(u,v)$ and $\mathbf{T}_v(u,v)$ associated to this parametrization.

   b. (5 points) Find a unit vector $\mathbf{n}(u,v)$ orthogonal to the surface at the point $\Phi(u,v)$.

   c. (10 points) Compute the area of the surface.

3. (25 points) Let C be the arc of the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$.

   a. (10 points) Write a parametrization $\mathbf{c}(t)$ that traces out the arc C for $-1 \le t \le 2$.

   b. (10 points) Compute the path integral $\int_{\mathbf{c}} \frac{y}{x}\,ds$.

   c. (5 points) Let $f(x,y) = \frac{e^x}{x^2+y^2}$. Compute the line integral $\int_{\mathbf{c}} (\nabla f)\cdot d\mathbf{s}$.

4. (25 points) Let S be the upper half of the unit sphere $x^2+y^2+z^2 = 1$ with $z \ge 0$. Suppose the surface is oriented so that the outer normal points upward, i.e., $\mathbf{n}\cdot\mathbf{k} > 0$. Find the flux $\int\int_S \mathbf{F}\cdot\,d\mathbf{S}$ of the vector field $\mathbf{F} = \frac{z}{2}\mathbf{k}$ through S.