Vector Calculus 20E, Spring 2012, Lecture B, Midterm 2

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the piece of the curve $y^2 = x^3$ which goes from $(0, 0)$ to $(1, 1)$. Calculate

$$\int_\gamma x^2 y dx - xy dy.$$

2. Let γ be the boundary of the standard square $[-1, 1] \times [-1, 1]$, oriented anticlockwise. Use Green's theorem to calculate

$$\int_\gamma (x - y^2) dx + (x^3 + y^4) dy.$$

3. Let Σ be the standard unit sphere, oriented using the outward normal, and let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (-x, y, z)$. Calculate (without using Gauss' theorem) the surface integral

$$\int_\Sigma \mathbf{F}.\mathbf{dS}$$

4. Let γ be the circle $x^2 + y^2 = 5$, oriented anticlockwise, and lying in the plane $z = 3$ inside $\mathbb{R}^3$. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (x^2 \cos x, y^2 \cos y, xyz)$. Use Stokes' theorem to calculate

$$\int_\gamma \mathbf{F}.\mathbf{dS}$$

Vector Calculus 20E, Spring 2013, Lecture A, Midterm 2

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the boundary of the square with vertices $(0,0), (1,0), (0,1), (1,1)$, oriented anticlockwise. Calculate

$$\int_\gamma (x^2 - y^2)dx + (x^2 + y^2)dy.$$

2. Let Σ be the unit upper hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$. Calculate

$$\int_\Sigma z^4 \, \mathrm{dA}.$$

3. Let Σ be the surface given by $x^2 + y^2 = 4$ and $-1 \leq z \leq 1$, oriented using the outward normal, and let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (x, y, z)$. Calculate

$$\int_\Sigma \mathbf{F}.\mathbf{dA}$$

4. Let Σ be the surface given by $y = 9 - x^2 - z^2$ and $y \geq 0$, with normal vector pointing in the direction of increasing y. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = (2xyz + 5z, \cos(yz), x^2y)$. Calculate

$$\int_\Sigma (\nabla \times \mathbf{F}).\mathbf{dA}$$

Vector Calculus 20E, Fall 2014, Lecture A, Midterm 2

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let Σ be the part of the paraboloid $z = x^2 + y^2$ lying between $z = 1$ and $z = 4$. Compute the surface area of Σ.

2. Let C be a circle of radius 1, lying in the plane $2x + 2y + z = 5$, and centred at the point $(1, 1, 1)$. Orient C clockwise, as seen from the origin. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (3\cos x + z, 5x - e^y, z^4 - 3y)$. Compute the circulation

$$\int_C \mathbf{F}.\mathrm{d}\mathbf{s}$$

3. Let Σ be the part of the hyperboloid $x^2 + y^2 = 1 + z^2$ lying between $z = -1$ and $z = +1$, oriented using normals pointing outward, away from the z-axis. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (x, y, z)$. Compute the flux

$$\int_\Sigma \mathbf{F}.\mathrm{d}\mathbf{S}$$

If you have any comments for me about the course so far (things you like, dislike, would like, etc.), please write them in the space below and hand this paper in at the end of the exam. You'll remain anonymous – I can't identify your handwriting!

(If you want to keep the questions, tear off the top part of this paper, but do it quietly!)

Vector Calculus 20E, Winter 2016, Lecture A, Midterm 2

50 minutes, 3 problems, no calculators allowed. Please start each problem on a new page.

You'll get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let Σ be the upper unit hemisphere (given by $x^2 + y^2 + z^2 = 1, z \geq 0$), oriented using the outward normal. Let $\mathbf{F}(x, y, z) = (y, x, z)$ be a vector field. Compute the flux

$$\iint_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

2. Let Σ be the part of the graph of the function $z = \cos(x^2+y^2)$ lying above the disc $x^2+y^2 \leq \pi^2$, oriented using the upward normal. Let $\mathbf{F}(x, y, z) = (-y, x, z)$ be a vector field. Compute the flux

$$\iint_\Sigma (\nabla \times \mathbf{F}) \cdot \mathrm{d}\mathbf{A}$$

3. The astroid curve $x^{2/3}+y^{2/3} = 1$ can be parametrised using $t \mapsto (\cos^3 t, \sin^3 t)$ with $0 \leq t \leq 2\pi$. Use Green's theorem to compute the area it encloses.

(Helpful identities: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\sin 2x = 2 \sin x \cos x$.)

Vector Calculus 20E, Winter 2016, Lecture A, Midterm 2

50 minutes, 3 problems, no calculators allowed. Please start each problem on a new page.

You'll get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let Σ be the upper unit hemisphere (given by $x^2 + y^2 + z^2 = 1, z \geq 0$), oriented using the outward normal. Let $\mathbf{F}(x, y, z) = (y, x, z)$ be a vector field. Compute the flux

$$\iint_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

2. Let Σ be the part of the graph of the function $z = \cos(x^2+y^2)$ lying above the disc $x^2+y^2 \leq \pi^2$, oriented using the upward normal. Let $\mathbf{F}(x, y, z) = (-y, x, z)$ be a vector field. Compute the flux

$$\iint_\Sigma (\nabla \times \mathbf{F}) \cdot \mathrm{d}\mathbf{A}$$

3. The astroid curve $x^{2/3}+y^{2/3} = 1$ can be parametrised using $t \mapsto (\cos^3 t, \sin^3 t)$ with $0 \leq t \leq 2\pi$. Use Green's theorem to compute the area it encloses.

(Helpful identities: $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\sin 2x = 2 \sin x \cos x$.)

Vector Calculus 20E, Winter 2017, Lecture B, Midterm 2

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Use Green's theorem to find the area of the region D of the plane whose lower boundary is the part of the x-axis where $0 \leq x \leq 2\pi$, and whose upper boundary is the curve C given by

$$t \mapsto (t - \sin t, 1 - \cos t), \qquad 0 \leq t \leq 2\pi.$$

2. Let Σ be the cone given by $z = 1 - \sqrt{x^2 + y^2}$ and $0 \leq z \leq 1$, oriented with the upper side considered positive. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (x, y, z)$. Compute the flux

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

3. Let Σ be the surface lying over the unit disc $x^2 + y^2 \leq 1$ and given by the formula $z = -3x^2 + 8xy + 3y^2$. Find the surface area of Σ.

Vector Calculus 20E, Winter 2017, Lecture B, Midterm 2

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Use Green's theorem to find the area of the region D of the plane whose lower boundary is the part of the x-axis where $0 \leq x \leq 2\pi$, and whose upper boundary is the curve C given by

$$t \mapsto (t - \sin t, 1 - \cos t), \qquad 0 \leq t \leq 2\pi.$$

2. Let Σ be the cone given by $z = 1 - \sqrt{x^2 + y^2}$ and $0 \leq z \leq 1$, oriented with the upper side considered positive. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (x, y, z)$. Compute the flux

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

3. Let Σ be the surface lying over the unit disc $x^2 + y^2 \leq 1$ and given by the formula $z = -3x^2 + 8xy + 3y^2$. Find the surface area of Σ.

Vector Calculus 20E, Fall 2017, Second Midterm

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the semicircle in $\mathbb{R}^2$ given by $x^2 + y^2 = 1$ and $x \geq 0$, oriented anticlockwise. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y) = (y^2, x)$. Compute

$$\int_\gamma \mathbf{F} \cdot \mathrm{d}\mathbf{s}.$$

2. Let Σ be the triangle in $\mathbb{R}^3$ whose corners are the points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$. Compute the integral

$$\int_\Sigma yz \; \mathrm{d}A.$$

3. Let Σ be the graph of the function $z = xy$, with $0 \leq x, y \leq 1$, oriented with the upward normal. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (1, x^2, xyz)$. Compute the flux

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

Vector Calculus 20E, Fall 2017, Second Midterm

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the semicircle in $\mathbb{R}^2$ given by $x^2 + y^2 = 1$ and $x \geq 0$, oriented anticlockwise. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y) = (y^2, x)$. Compute

$$\int_\gamma \mathbf{F} \cdot \mathrm{d}\mathbf{s}.$$

2. Let Σ be the triangle in $\mathbb{R}^3$ whose corners are the points $(1, 0, 0), (0, 1, 0), (0, 0, 1)$. Compute the integral

$$\int_\Sigma yz \; \mathrm{d}A.$$

3. Let Σ be the graph of the function $z = xy$, with $0 \leq x, y \leq 1$, oriented with the upward normal. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x, y, z) = (1, x^2, xyz)$. Compute the flux

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$