Vector Calculus 20E, Spring 2012, Lecture B, Midterm 1

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Calculate the integral of $f(x, y) = xe^{-y}$ over the triangle in $\mathbb{R}^2$ formed by the lines $x = 0$, $y = 0$, and $x + y = 1$.

2. Calculate the integral of the function $f(x, y, z) = z$ along the curve given by $(t, \frac{2}{3}t^{\frac{3}{2}}, t)$, where $2 \leq t \leq 7$.

3. Let $D^* = \{(u, v) : u^2 + v^2 \leq 1\}$ and let D be the image of D^* under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $x = u^3, y = v^3$. Calculate the area of D. (Hint: the following formulae may be useful.)

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

4. Use spherical polar coordinates to calculate the integral of the function $f(x, y, z) = z^2$ over the region of $\mathbb{R}^3$ between the spheres of radius 1 and 2.

Vector Calculus 20E, Spring 2013, Lecture A, Midterm 1

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

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Simplify answers if you can, but don't worry if you can't!

1. Let $g : \mathbb{R}^3 \to \mathbb{R}^2$ be a map such that $g(0, 0, 0) = (2, 3)$ and whose derivative Dg at $(0, 0, 0)$ is given by the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = xy - x + y$. Calculate the derivative $D(f \circ g)$ at $(0, 0, 0)$.

2. Let D be the region between the graphs of $y = x^2$ and $y = x^3$. Compute the integral

$$\iint_D xy \,\mathrm{d}A.$$

3. Let $D^* = \{(u, v) : 0 \leq u \leq 1,\ 0 \leq v \leq 1\}$ and let D be the image of D^* under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(u, v) = (u^2 v, uv^2)$. Calculate the area of D.

4. Let $f(x, y) = x \log y$. Find the approximation to the value of $f(1.03, 1.02)$ given by the second-order Taylor expansion of f at $(1, 1)$.

Vector Calculus 20E, Spring 2014, Lecture A, Midterm 1

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let H be the upper unit hemisphere given by $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$. Calculate

$$\iiint_H z^2 \, dV.$$

2. Let D be the triangle whose vertices are the points $(0,1), (1,0)$ and $(2,0)$. Calculate

$$\iint_D xy \, dA.$$

3. Compute the second-order Taylor approximation, at the point $(1,1)$, of the function

$$f(x,y) = \frac{1}{x^2 + y^2}.$$

4. Let D^* be the unit square given by $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Let D be the image of D^* under the map $T(u,v) = (e^{u+v}, e^{u-v})$. Calculate the area of D.

If you have any comments for me about the course so far (things you like, dislike, would like, etc.), please write them in the space above, tear it off and hand it in at the end of the exam. (I don't know what your writing looks like, so you'll remain anonymous!)

Vector Calculus 20E, Winter 2016, Lecture A, Midterm 1

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

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Simplify answers if you can, but don't worry if you can't!

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (xy, x + y^2)$. Let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be the composite function $g = f \circ f$, so that $g(x, y) = f(f(x, y))$. Compute the matrix of the derivative Dg at the point $(1, 2)$.

2. Compute the second-order Taylor approximation, at the point $(1, \frac{\pi}{2})$, to the function $f(x, y) = \sin(x^2 y)$: express your answer in the form

$$f(1 + h_1, \frac{\pi}{2} + h_2) = \dots.$$

3. Let D be the region in $\mathbb{R}^2$ where $x \geq 0, y \geq 0$ and $x^{1/3} + y^{1/3} \leq 1$. Use the change of variables $x = u^3, y = v^3$ to calculate the area of D.

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Vector Calculus 20E, Winter 2017, Lecture B, Midterm 1

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let D be the region between the graphs of $y = x^2$ and $y = 2 - x^2$. Compute

$$\int_D x^2 y \, dxdy$$

2. Let $f(x, y) = e^{3x-2y}$. Compute the second-order Taylor approximation to f at the point $(1, 1)$, expressing your answer in the form

$$f(1 + h_1, 1 + h_2) = \cdots .$$

3. Let D be the region bounded by the lines

$$x + y = 0 \qquad x + y = 2 \qquad x - y = 0 \qquad x - y = 2.$$

Compute

$$\int_D (x + y)e^{x^2-y^2} \, dxdy$$

by using the change of variables $u = x + y,\ v = x - y$.

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Vector Calculus 20E, Fall 2017, Lecture B, Midterm 1

Fifty minutes, three problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let D be the upper half of the unit disc, given by $x^2 + y^2 \leq 1$ and $y \geq 0$. Find the average value on D of the function $f(x, y) = y$.

2. Let $f(x, y) = \sin(xy)$. Compute the second-order Taylor approximation to f at the point $(1, \frac{\pi}{2})$, expressing your answer in the form

$$f(1 + h_1, \frac{\pi}{2} + h_2) = \cdots .$$

3. Let D be the parallelogram with vertices $(0, 0), (2, 0), (1, 1), (3, 1)$. Compute

$$\int_D (xy - y^2)\ dxdy$$

by changing variables using $x = 2u + v$ and $y = v$.

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