Math 20E - Lecture A00 | **Final Exam, VERSION A**
Fall 2016 | **Name:** ____________
12/06/2016 | PID: ____________
Time Limit: 3 Hours | Section Time: ____________

This exam contains 2 pages (including this cover page) and 10 questions.
Total of points is 100.

You may not use any notes (except your cheat sheet) or calculators during this exam.
Write your *Name, PID, and Section* on the front of your Blue Book.
Write the *Version* of your exam on the front of your Blue Book.
Write your solutions clearly in your Blue Book.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.

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1. (3 points) Carefully read and complete the instructions at the top of this exam sheet. Just give all 3 points.

2. (10 points) Using Green's theorem, or otherwise, to evaluate $\int_C x^3\,dy - y^3\,dx$, where C is the unit circle $x^2 + y^2 = 1$ with the counterclockwise orientation.

3. (20 points) Let $\mathbf{F}(x, y, z) = (3x^2 + 2xy)\mathbf{i} + (2yz^2 + x^2 + 3y)\mathbf{j} + (2y^2z)\mathbf{k}$.

   a. (5 points) Compute $\nabla \cdot \mathbf{F}$.

   b. (5 points) Compute $\nabla \times \mathbf{F}$.

   c. (5 points) Is $\mathbf{F}$ conservative? If yes, find the function f such that $\mathbf{F} = \nabla f$.

   d. (5 points) Let $\mathbf{c}(t) = (\cos t, \sin t, 0)$ for $0 \le t \le \pi$. Compute the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$.

4. (10 points) Let S be the hemisphere $x^2 + y^2 + z^2 = 9, z \ge 0$, oriented with the upward normal. Let $\mathbf{F}$ be the vector field $(x^2 + z)\mathbf{i} + 3z(e^{x^2+y^2} + x^{10})\mathbf{j} + 2y^{15}z\mathbf{k}$. Compute the integral $\int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

5. (10 points) Find the flux of the vector field $\mathbf{F}(x, y, z) = x^3y\mathbf{i} + z^8\mathbf{j} - 3x^2yz\mathbf{k}$ out of the surface of the standard unit cube $(0 \le x, y, z \le 1)$ in $\mathbb{R}^3$. (Hint: use Gauss' Theorem)

6. (10 points) Let $\mathbf{c}$ be the path given by $\mathbf{c}(t) = (t, \cos t, \sin t)$ for $0 \le t \le \frac{\pi}{2}$. Find the length of the path.

7. (10 points) Evaluate the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y, z) = (-y, x, e^{-z})$ and $\mathbf{c}(t) = (\cos t, \sin t, t)$ for $0 \le t \le 2\pi$.

8. (10 points) Let S be the surface $z = x^2 + y^2, 0 \leq z \leq 4$. Evaluate the following integral $\int\int_S \frac{y}{\sqrt{z}}\, dS$.

9. (10 points) Let S be the part of the cone $z = \sqrt{x^2 + y^2}$ lying above the standard unit square $0 \leq x, y \leq 1$. Compute the surface area of S.

10. (7 points) Let S be a surface in $\mathbb{R}^3$, and let ∂S be the boundary of S. Let $\mathbf{F}$ be a vector field on S with continuous partial derivatives. Suppose that you are given the following information about S and $\mathbf{F}$:

    i. S lies in the plane $y = 3$.
    ii. Area(S) $= 17$.
    iii. Length(∂S) $= 25$.
    iv. $\text{div}(\mathbf{F}) = x^2 + y^2 - z$.
    v. $\text{curl}(\mathbf{F}) = 3x\mathbf{i} - y\mathbf{j} - 2z\mathbf{k}$.

    Using this information, evaluate the absolute value of the line integral $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.