Vector Calculus 20E, Spring 2013, Lecture A, Final exam

Three hours, eight problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the closed curve given by the equations $x = t^2 - t,\ y = 2t^3 - 3t^2 + t$ for $0 \le t \le 1$. Using Green's theorem, find the area enclosed by the curve γ.

2. Find the integral $\int_\gamma \mathbf{F}.\mathbf{ds}$ where $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and γ is the helical curve $x = 2\cos t,\ y = 2\sin t,\ z = t$ for $0 \le t \le 2\pi$, oriented in the direction of increasing t.

3. Find the integral $\int_\Sigma y^2 \,\mathrm{dA}$, where Σ is the part of the cylinder $x^2 + y^2 = 4$ lying between the planes $z = 0$ and $z = x + 3$.

4. Let D be the standard unit disc in the xy-plane, and let Σ be the part of the graph of the function $z = xy$ lying over the domain D. Find the surface area of Σ.

5. Let Σ be the hemisphere $x^2 + y^2 + z^2 = 16, z \ge 0$, oriented with the upward normal, and let Let $\mathbf{F}$ be the vector field $(x^2 + z)\mathbf{i} + 3xyz\mathbf{j} + (2xz)\mathbf{k}$. Compute the integral $\int_\Sigma (\nabla \times \mathbf{F}).\mathbf{dA}$

6. Find the flux of the vector field $\mathbf{F} = x^2y\mathbf{i} + z^8\mathbf{j} - 2xyz\mathbf{k}$ out of the surface of the standard unit cube $(0 \le x, y, z \le 1)$ in $\mathbb{R}^3$.

7. Find the integral $\int_\gamma \mathbf{F}.\mathbf{ds}$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and γ is the oriented curve given by

$$(\sin^2 t \cos t, \cos^2 t \sin t, (t - \pi)^4) \qquad 0 \le t \le 2\pi.$$

8. One of the two vector fields

$$\mathbf{F} = 3x^2y\mathbf{i} + x^3\mathbf{j} + 5\mathbf{k}$$
$$\mathbf{G} = (x + z)\mathbf{i} + (z - y)\mathbf{j} + (x - y)\mathbf{k}$$

is conservative, and the other is not. Which is which? Find a potential for the conservative one.