Vector Calculus 20E, Spring 2012, Lecture B, Final exam

Three hours, eight problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the ellipse $x^2 + 4y^2 = 4$, oriented anticlockwise. Compute

$$\int_\gamma (4y - 3x)dx + (x - 4y)dy$$

2. Find the integral $\int_\gamma \mathbf{F}.d\mathbf{s}$ where $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and the curve γ is the part of the parabola $z = x^2, y = 0$ going from $x = -1$ to $x = 2$.

3. Find the integral $\int_R xyz\, dS$, where R is the rectangle in $\mathbb{R}^3$ whose vertices are the points $(0,0,0), (1,0,0), (0,1,1), (1,1,1)$.

4. Find the area of the surface Σ in $\mathbb{R}^3$ described by

$$(u\cos v, u\sin v, u^2) \qquad 0 \le u \le 2 \quad 0 \le v \le 2\pi.$$

5. Find the flux $\int_\Sigma \mathbf{F}.d\mathbf{S}$ of the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z^3\mathbf{k}$ through the surface Σ in $\mathbb{R}^3$ which is oriented with an upward normal vector and described by

$$(u\cos v, u\sin v, v) \qquad 0 \le u \le 2 \quad 0 \le v \le 2\pi.$$

6. Find the flux of the vector field $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ out of the unit sphere in $\mathbb{R}^3$.

7. Find the integral $\int_\gamma \mathbf{F}.d\mathbf{s}$ where $\mathbf{F} = x\mathbf{i} + y^2\mathbf{j} + z^3\mathbf{k}$ and γ is the oriented curve given by

$$(\sin^2 t, \cos^3 t, \sin^4 t) \qquad 0 \le t \le 2\pi.$$

8. One of the two vector fields

$$\mathbf{F} = y^2\mathbf{i} - z^2\mathbf{j} + x^2\mathbf{k}$$
$$\mathbf{G} = (x^3 - 3xy^2)\mathbf{i} + (y^3 - 3x^2y)\mathbf{j} + z\mathbf{k}$$

is conservative, and the other is not. Which is which? Find a potential for the conservative one.