Math 152: Topics in Data Science

Thang Huynh, UC San Diego

Contact Information

Instructor: Thang Huynh

Email: tlh007@ucsd.edu

Office Hours: 9:00am-10:30am Wednesday at AP&M 6341
(or by appointment)

Course webpage: Syllabus (must read), Exam Schedule, Homework, TA Information, etc.

`www.thanghuynh.io/teaching/math152_winter19/home/`

What is this course about?

- **Prerequisite:** I highly recommend that students are familiar with linear algebra (MATH 102, recommended), probability theory (MATH 180A), and combinatorics. The class will attempt to be self contained (but this is not always possible). Moreover, the class is theoretical, and is devoted to ideas, algorithms, and proofs.

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- **We will cover among other topics (tentative)**: sampling, finding frequent items, counting distinct elements, general frequency moment estimation, dimensionality reduction, and matrix approximation.

Course Information

There is no course textbook.

- *Data Stream Algorithms* by Amit Chakrabarti.
- *Mining of Massive Datasets* by Jure Leskovec, Anand Rajaraman, Jeff Ullman.
- *Foundations of Data Science* by Avrim Blum, John Hopcroft, and Ravindran Kannan.
- *Matrix Methods in Data Mining and Pattern Recognition* by Lars Elden.

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- Exams: There will be two midterm exams and a final exam. You may use one 8.5 x 11 inch page of handwritten notes.
- There will be no makeup exams.

Plan

Week	Contents
1	Linear Algera Review
2	Probability Review
3	Sampling
4	Finding frequent items
5	Random Projections
6	Random Projections
7	Singular Value Decomposition
8	Matrix sampling
9	Matrix approximation
10	Nearest neighbor search
11	Final Exam

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- The above two processes are dimensionality reduction.

Dimensionality reduction: A cool example

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See `http://www.learnopencv.com/principal-component-analysis/`

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We can ask a single question:
"Are you a democrat or a republican?"

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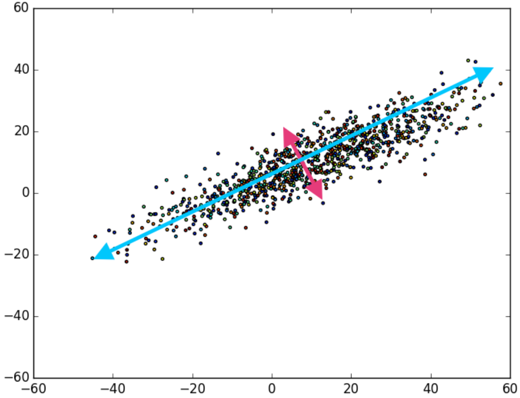
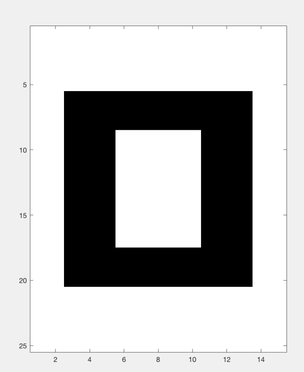
Principal Component Analysis (PCA)



Thanks to Austin G. Walters

Data compression using singular value decompositions

We want transmit the following image, which consists of an array of 15×25 black or white pixels.



See `http://www.ams.org/publicoutreach/feature-column/fcarc-svd`

Data compression using singular value decompositions

We will represent the image as a 15×25 matrix in which each entry is either a 0, representing a black pixel, or 1, representing white (375 entries).

```
A =

     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     1     1     1     1     1     0     0     0     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     0     0     0     0     0     0     0     0     0     0     0     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
```

Data compression using singular value decompositions

Using SVD, we can represent A as

$$A = \boldsymbol{u}_1 \sigma_1 \boldsymbol{v}_1^T + \boldsymbol{u}_2 \sigma_2 \boldsymbol{v}_2^T + \boldsymbol{u}_3 \sigma_3 \boldsymbol{v}_3^T$$

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- Each $\boldsymbol{v}_i$ has 15 entries,
- each $\boldsymbol{u}_i$ has 25 entries, and
- there are three singular values σ_i.

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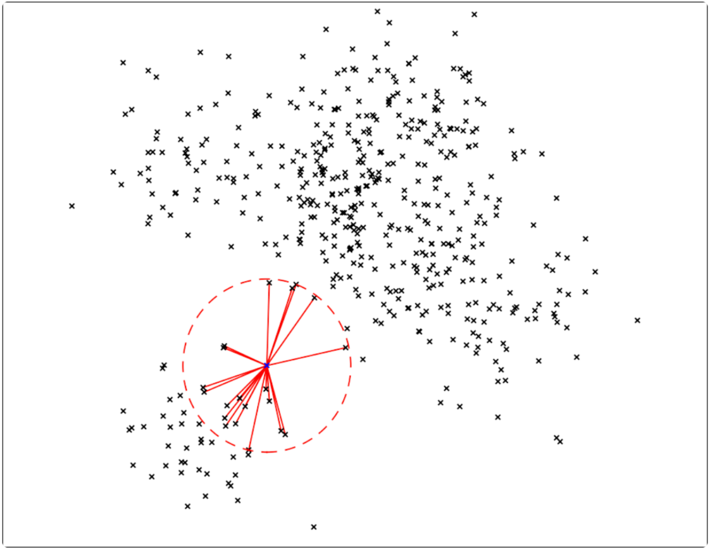
- there are three singular values σ_i.

This implies that we may represent the matrix using only 123 numbers rather than the 375 that appear in the matrix, and still preserve all the information of the matrix A.

Nearest neighbor search



Counting distinct elements

How many distinct IP addresses has the router seen? (An IP may have passed once, or many many times.)

Web searchers

Millions of queries / day

- What are the top queries right now?
- Which terms are gaining popularity now?
- What ads should we show for this query and user?