Name: **PID:**

- Print your *NAME* on every page and write your PID in the space provided above.
- Show all of your work in the spaces provided. No credit will be given for unsupported answers, even if correct.
- Supporting work for a problem must be on the page containing that problem. No scratch paper will be accepted.
- No calculators, tables, phones, or other electronic devices are allowed during this exam. You may use your double-sided handwritten notes, but no books or other assistance.

DO NOT TURN PAGE UNTIL INSTRUCTED TO DO SO
(This exam is worth 25 points)

Problem 0.(1 point.) Follows the instructions on this exam and any additional instructions given during the exam.

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Problem 1.(6 points.) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Find the SVD of A.

(b) Run the power method starting from $x^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ for $k = 3$ steps.

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Problem 2.(6 points.) Suppose that a matrix A has the following SVD

$$A = [u_1\ u_2\ u_3]\Sigma \begin{bmatrix} -v_1^T- \\ -v_2^T- \\ -v_3^T- \end{bmatrix} = \begin{bmatrix} .13 & .02 & -.01 \\ .41 & .07 & -.03 \\ .55 & .09 & .-0.4 \\ .68 & .11 & -.05 \\ .15 & -.59 & .65 \\ .07 & -.73 & -.67 \\ .07 & -.29 & .32 \end{bmatrix} \begin{bmatrix} 12.4 & 0 & 0 \\ 0 & 9.5 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \begin{bmatrix} .56 & .59 & .56 & .09 & .09 \\ .12 & -.02 & .12 & -.69 & -.69 \\ .40 & -.80 & .40 & .09 & .09 \end{bmatrix}.$$

Let $\sigma_1 = 12.4, \sigma_2 = 9.5$, and $\sigma_3 = 1.3$ be the singular values of A. Let $A_2 = \sum_{i=1}^{2} \sigma_i u_i v_i^T$.

(a) Express $\|A_2\|_F^2$ and $\|A - A_2\|_2^2$ in term of singular values of A. (You need not to simplify.)

(b) What is the best rank-1 approximation matrix to A (in Frobenius norm)?

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Problem 3.(6 points.) Describe the process of estimating F_0 (or counting distinct elements in a data stream.) that you learned in class.

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Problem 4.(6 points.) Suppose there is a random variable X taking values in $[0, 1]$. Note that we don't know the distribution of X. How can you estimate $\mathbb{E}[X]$ up to an error 0.1 and with probability at least 90%? In this case, how many samples do you need to take? Explain your answer clearly. (You need not to simplify.)